

Chapter 13. Nuclei

Radioactivity and Decay Law

1 Mark Questions

1. Why is it found experimentally difficult to detect neutrinos in nuclear β -decay?
[All India 2014]

Ans. Neutrinos are difficult to detect because they are massless, have no charge and do not interact with nucleons.

2. Define the activity of a given radioactive substance. Write its SI unit. [All India 2013]

Ans. The activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

SI unit of activity is Becquerel (Bq).

1 Bq = 1 disintegration/second

3. Write any two characteristic properties of nuclear force. [All India 2011]

Ans. Two characteristics of nuclear force are given as below:

- (i) These are short range forces.
- (ii) These are strong force of attractive nature

4. How is the mean life of a radioactive sample related to its half-life? [Foreign 2011]

Ans.

Mean life, $t_m = \frac{1}{\lambda}$, where λ is decay constant.

But half-life, $T_{1/2} = \frac{\ln 2}{\lambda}$

$$\Rightarrow \lambda = \frac{\ln 2}{T_{1/2}}$$

$$\Rightarrow t_m = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693}$$

$$t_m = 1.443 T_{1/2}$$

5. How is the radius of a nucleus related to its mass number? [All India 2011 c]

Ans.

Radius of nucleus, $R = R_0 A^{1/3}$

where, R = radius of nucleus

A = mass number

$$R_0 = 1.2 \text{ fm} = 1.2 \times 10^{-15} \text{ m}$$

6. A nucleus undergoes β -decay. How does its

- (i) mass number
- (ii) atomic number change? [Delhi 2011C]

Ans.

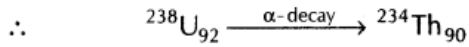
During β -decay,

- (i) no change in mass number.
- (ii) atomic number increases by 1.

7. A nucleus ${}^{238}_{92}\text{U}$ undergoes α -decay and transforms to thorium. What is
 (i) the mass number and
 (ii) atomic number of the nucleus produced? [Delhi 2011C]

Ans.

In α -decay, the mass number of parent nucleus decreases by 4 units and atomic number decreases by 2 units.



- (i) Mass number of the nucleus produced
 = 234 (1/2)
 (ii) Atomic number of nucleus produced
 = 90 (1/2)

8. Two nuclei have mass numbers in the ratio 1: 8. What is the ratio of their nuclear radii? [All India 2009]

Ans.

Radius of nucleus, $R = R_0 A^{1/3}$

where, R_0 = constant, A = mass number

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$$

$$R_1 : R_2 = 1 : 2$$

9. Two nuclei have mass numbers in the ratio 8:125. What is the ratio of their nuclear radii? [All India 2009]

Ans. Refer to ans. 8. (Ans. 2 : 5)

10. Two nuclei have mass numbers in the ratio 27 : 125. What is the ratio of their nuclear radii? [All India 2009]

Ans. Refer to ans. 8. (Ans. 3 : 5)

11. Two nuclei have mass numbers in the ratio 1:2. What is the ratio of their nuclear densities? [Delhi 2009]

Ans. Nuclear density is independent of mass number.

12. Assuming the nuclei to be spherical in shape, how does the surface area of a nucleus of mass number A_1 compare with that of a nucleus of mass number A_2 ? [All India 2008 C]

Ans.

Radius of nucleus, $R = R_0 A^{1/3}$

where, $R_0 = \text{constant}$

\therefore Surface area, $S = 4\pi R^2$

$$= 4\pi (R_0 A^{1/3})^2 = 4\pi R_0^2 A^{2/3}$$

\therefore Ratio of surface areas, $\frac{S_1}{S_2} = \left(\frac{A_1}{A_2}\right)^{2/3}$

13. Out of the two characteristics, the mass number A and the atomic number Z of a nucleus, which one does not change during nuclear decay? [All India 2008 C]

Ans.

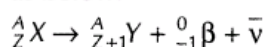
The mass number A of a nucleus does not change during nuclear β -decay. (1)

2 Marks Questions

14. In both β^- and β^+ -decay process, the mass number of nucleus remains the same, whereas the atomic number Z increases by one in β^- -decay and decrease by one in β^+ -decay. Explain giving reasons. [Foreign 2014]

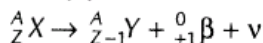
Ans.

In β^- -decay, a β -particle of zero mass and -1 charge is emitted. The decay process is shown as below:



Since, the mass of β^- -particle is negligibly small and the atomic number increases by 1 due to the loss of 1 negative charge. (1)

Similarly, for a β^+ -decay, a β -particle of negligibly small and $+1$ charge is emitted. The decay process is shown as below:



The mass number remains the same but here the atomic number decreases by 1 due to the loss of 1 positive charge. (1)

15. In a given sample, two radio isotopes A and B are initially present in the ratio of 1 : 4. The half-lives of A and B are 100 yr and 50 yr, respectively. Find the time after which the amounts of A and B become equal. [HOTS; Foreign 2012]

Ans.



In these types of questions, we have to keep in mind the exponential decay.

Let N_A be the concentration of A after time t_A and N_B be the concentration of B after time t_B .
From radioactive disintegration equation,

$$N_A = N_0 e^{-\lambda_A t_A}$$

$$N_B = 4N_0 e^{-\lambda_B t_B} \quad [As, N_{0B} = 4N_{0A}]$$

Now, half-life of A is 100 yr and B is 50 yr.

$$\text{So, } \lambda_A = \frac{\ln 2}{100} \text{ and } \lambda_B = \frac{\ln 2}{50}$$

Dividing, we get

$$\frac{\lambda_A}{\lambda_B} = \frac{1}{2} \text{ or } \lambda_B = 2\lambda_A \quad (1)$$

Let after t years, $N_A = N_B$

$$\text{So, } \frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{4e^{-\lambda_B t}} \quad N_A = N_B$$

$$\Rightarrow 4e^{-\lambda_B t} = e^{-\lambda_A t}$$

$$\Rightarrow 4 = e^{-(\lambda_A - \lambda_B)t}$$

$$\ln 4 = -(\lambda_A - 2\lambda_A)t \quad [\because \lambda_B = 2\lambda_A]$$

$$\ln 4 = \lambda_A t$$

$$t = \frac{\ln 4}{\ln 2} \times 100 = 200 \text{ yr} \quad \left[\because \lambda_A = \frac{\ln 2}{100} \right] \quad (1)$$

16. How the size of a nucleus is experimentally determined? Write the relation between the radius and mass number of the nucleus. Show that the density of nucleus is independent of its mass number.

[Delhi, 2012, 2011 C]

Ans.



The size of nucleus will increase with the increase of mass number.

The size of the nucleus is experimentally determined using Rutherford's α -scattering experiment and the distance of closest approach and impact parameter.

The relation between radius and mass number of nucleus is

$$R = R_0 A^{1/3}, \text{ where } R_0 = 1.2 \text{ fm}$$

where, A = mass number, R = radius of nucleus

(1)

Nuclear density,

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi(R_0 A^{1/3})^3}$$

where, m = mass of each nucleon

$$\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} \Rightarrow \rho = \frac{m}{\frac{4}{3}\pi R_0^3}$$

From the above formula, it is clear that ρ does not depend on the mass number.

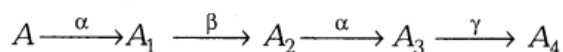
(1)

17. In β -decay, the experimental detection of neutrino is found to be difficult.
[Delhi, 2012, 2011C]



Ans.

18. A radioactive nucleus A undergoes a series of decays according to the following scheme

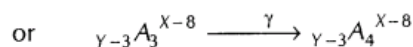
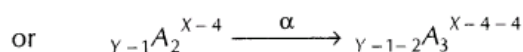
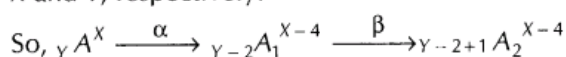


The mass number and atomic number of A_4 are 172 and 69, respectively. What are these numbers for A? [Delhi 2009]

Ans.

In α -decay, the atomic number decreases by 2 units and mass number decreases by 4 units. In β -decay, the atomic number increases by 1 unit but mass number does not change. In γ -decay, there is no change in atomic number and mass number. (1)

Let the mass number and atomic number of A be X and Y, respectively.



According to the question, the mass number and atomic number of A_4 are 172 and 69.

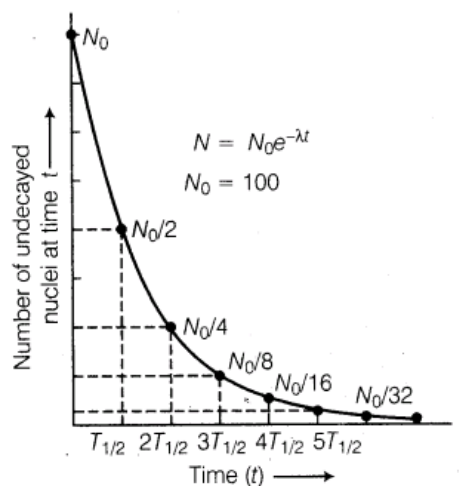
$$\therefore X - 8 = 172 \Rightarrow X = 172 + 8 = 180$$

$$Y - 3 = 69 \Rightarrow Y = 72 \quad (1)$$

19. Draw a plot representing the law of radioactive decay. Define the activity of a sample of a radioactive nucleus. Write its SI unit. [Foreign 2008]

Ans.

The curve representing the law of radioactive decay is shown as below:



Decay curve for a radioactive element (1)

The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

The SI unit of activity is Becquerel (Bq).

$$1 \text{ Bq} = 1 \text{ disintegration/second} \quad (1)$$

- 20.** A radio nuclide sample has N_0 nuclei at $t = 0$. Its number of undecayed nuclei get reduced to $\frac{N_0}{e}$ at $t = \tau$. What does the term τ stand for? Write in terms of τ the time interval T in which half of the original number of nuclei of this radio nuclide would have got decayed.

[Delhi 2008C]

Ans.

- (i) The term τ stands for mean life.
 (ii) The required relation is $\tau = 1.44T$.
 i.e. mean life of radioactive sample
 $= 1.44 \times \text{half-life}$

3 Marks Questions

- 21.** (i) Deduce the expression, $N = N_0 e^{-\lambda t}$ for the law of radioactive decay.
 (ii) (a) Write symbolically the process expressing the β^+ - decay of ${}^{22}_{11}\text{Na}$. Also, write the basic nuclear process underlying this decay.
 (iii) Is the nucleus formed in the decay of the nucleus ${}^{22}_{11}\text{Na}$ is isotope or isobar?

[Delhi 2014]

Ans.

- (i) Rutherford and Soddy made experimental study of the radioactive decay of various radioactive materials and found that the decay of all radioactive materials is governed by the same general law.

According to this law, the rate of decay of radioactive atoms at any instant is proportional to the number of atoms present at that instant.

Let N be the number of atoms present in a radioactive substance at any instant t . Let dN be the number of atoms that disintegrate in a short interval dt . Then, the rate of disintegration $-dN/dt$ is proportional to N , i.e.

$$-\frac{dN}{dt} = \lambda N$$

where, λ is a constant for the given substance and is called **decay constant** (or disintegration constant or radioactive constant or transformation constant). For a given element, the value of λ is constant but for different elements it is different. From the above equation, we have

$$\frac{dN}{N} = -\lambda dt \quad (1)$$

On integrating both sides, we get

$$\log_e N = -\lambda t + C \quad \dots(i)$$

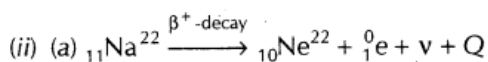
where, C is the integration constant. To determine C , we apply the initial conditions. Suppose, there were N_0 atoms in the beginning, i.e., $N = N_0$ at $t = 0$.

$$\text{Then, } \log_e N_0 = C$$

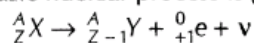
Substituting this value of C in the above Eq.(i), we have

$$\begin{aligned} \log_e N &= -\lambda t + \log_e N_0 \\ \log_e N - \log_e N_0 &= -\lambda t \text{ or } \log_e \frac{N}{N_0} = -\lambda t \\ \frac{N}{N_0} &= e^{-\lambda t} \Rightarrow N = N_0 e^{-\lambda t} \quad \dots(ii) \end{aligned}$$

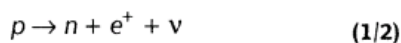
Here, N_0 and N are numbers of atoms in a radioactive substance at time $t = 0$ and after time t respectively. (1)



The basic nuclear process is given by



For β^+ -decay, there is a conversion of a proton into neutron to emit positron (positive electron).



(iii) The nucleus formed in the decay is an isobar. (1/2)

22. (i) Define the terms (a) half-life (b) average life. Find out the relationship with the decay constant (λ).

(ii) A radioactive nucleus has a decay constant $\lambda = 0.3465 \text{ (day)}^{-1}$. How long would it take the nucleus of decay to 75% of its initial amount? **[Foreign 2014]**

Ans.

- (i) (a) **Half-life** Half-life of a radioactive element is defined as the time during which half the number of atoms present initially in the sample of the element decay or it is the time during which number of atoms left undecayed in the sample is half the total number of atoms present in the sample. It is represented by $T_{1/2}$.

From the equation $N = N_0 e^{-\lambda t}$,

$$\text{At half-life, } t = T_{1/2}, N = \frac{N_0}{2}$$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2$$

On taking log on both sides, we get

$$\lambda T_{1/2} = \log_e 2$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{\log_{10} 2 \times 2.303}{\lambda}$$

$$= \frac{0.3010 \times 2.303}{\lambda}$$

After n half-life, the the number of atoms

left undecayed is given by $N = N_0 \left(\frac{1}{2}\right)^n$

$$T_{1/2} = \frac{0.6932}{\lambda} \quad (1)$$

- (b) **Average Life** Average life of a radioactive element can be obtained by calculating the total life time of all atoms of the element and dividing it by the total number of atoms present initially in the sample of the element.

Average life or mean life of radioactive element is

$$\tau = \frac{\text{Total life time of all atoms}}{\text{Total number of atoms}}$$

$$\tau = \int_0^{N_0} \frac{t dN}{N_0} = \int_0^\infty \frac{-\lambda N_0 e^{-\lambda t} dt \times t}{N_0}$$

[When $N = N_0, t = 0$ and when $N = 0, t = \infty$
[$\because dN = -\lambda(N_0 e^{-\lambda t}) dt$]

$$= \lambda \int_0^\infty t e^{-\lambda t} dt = \lambda \left[\left\{ t \frac{e^{-\lambda t}}{-\lambda} \right\}_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$= \lambda \left(0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right)$$

$$= \int_0^\infty e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^\infty = 0 - \frac{1}{-\lambda} = \frac{1}{\lambda}$$

$$\tau = \frac{1}{\lambda} = \frac{1}{0.6931/T_{1/2}}$$

$$\tau = 1.44 T_{1/2} \quad (1)$$

- (ii) Given, $\lambda = 0.3465 \text{ (day)}^{-1}$

According to the radioactive decay law, we have

$$R = R_0 e^{-\lambda t}$$

$$\Rightarrow \frac{R_0 \times 75}{100} = R_0 e^{-0.3465t} \Rightarrow \frac{3}{4} = e^{-0.3465t}$$

$$\Rightarrow t = 0.830 \text{ s} \quad (1)$$

- 23.** (i) Define the term activity of a sample of radioactive nucleus. Write its SI unit.
- (ii) The half-life of ${}_{92}^{238}\text{U}$ undergoing α -decay is 4.5×10^9 yr. Determine the activity of 10g sample of ${}_{92}^{238}\text{U}$. Given that 1g of ${}_{92}^{238}\text{U}$ contains 25.3×10^{20} atoms. [All India 2014 C]

Ans.

- (i) The activity of a sample of radioactive nucleus equals to its decay rate (or number of nuclei decaying per unit time). Its SI unit is disintegrations or Becquerel. $\left(1 \frac{1}{2}\right)$

- (ii) Given,

Number of atoms in 1 g of ${}_{92}^{238}\text{U}$ is 25.3×10^{20} atoms.

From the radioactive decay,

$$\frac{dN}{dt} \propto N \Rightarrow R \propto N \quad \left[\because \frac{dN}{dt} = R \right]$$

$$R = \lambda N = \frac{\log_e 2 \times 25.3 \times 10^{20} \times 10}{4.5 \times 10^9}$$

$$= \frac{0.6931 \times 25.3 \times 10^{21}}{4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60}$$

$$= 1.24 \times 10^5 \text{ dps} \quad \left(1 \frac{1}{2}\right)$$

- 24.** (i) The number of nuclei of a given radioactive sample at time $t = 0$ and $t = T$ are N_0 and N_0/n , respectively. Obtain an expression for the half-life ($T_{1/2}$) of the nucleus in terms of n and T .
- (ii) Write the basic nuclear process underlying β -decay of a given radioactive nucleus. [Delhi 2013C]

Ans.

- (i) According to the law of radioactive decay,

$$N = N_0 e^{-\lambda t}$$

$$N = \frac{N_0}{n} \text{ and } t = T$$

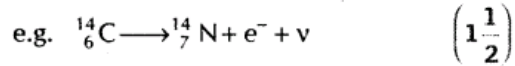
$$\therefore \frac{N_0}{n} = N_0 e^{-\lambda T}$$

$$\Rightarrow n = e^{\lambda T}$$

$$\Rightarrow \lambda = \frac{\log n}{T}$$

$$\therefore \text{Half-life, } T_{1/2} = \frac{0.6931}{\lambda} = \frac{0.693T}{\log n} \quad \left(1 \frac{1}{2}\right)$$

(ii) In β^- -decay process, a nucleus emits a negative charge. A neutron is converted to a proton causing the nuclide's atomic number to increase by one but the atomic mass remains the same.



25. State the law of radioactive decay.

Plot a graph showing the number N of undecayed nuclei as a function of time t -for a given radioactive sample having half-life $T_{1/2}$.

Depict in the plot, the number of undecayed nuclei at

(i) $t = 3 T_{1/2}$ (ii) $t = 5 T_{1/2}$. [Delhi 2011]

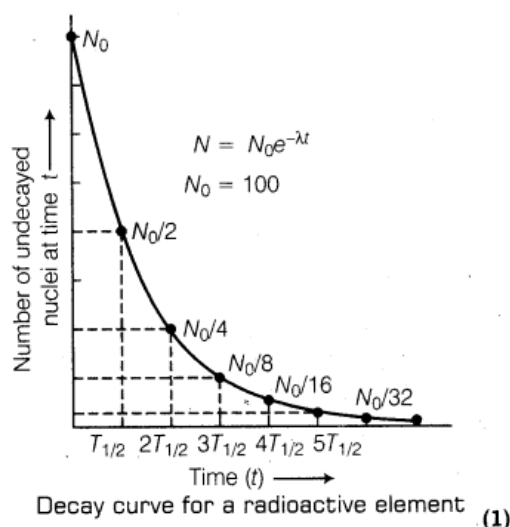
Ans.

Law of Radioactive Decay The rate of disintegration of radioactive sample at any instant is directly proportional to the number of undisintegrated nuclei present in the sample at that instant, i.e.

$$\frac{dN}{dt} \propto N \quad \frac{dN}{dt} = -\lambda N \quad (1)$$

where, N = number of undisintegrated nuclei present in the sample at any instant t and $\frac{dN}{dt}$ is rate of disintegration. (1)

The curve representing the law of radioactive decay is shown as below:

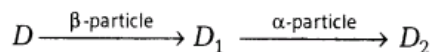


The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

The SI unit of activity is Becquerel (Bq).

$$1 \text{ Bq} = 1 \text{ disintegration/second} \quad (1)$$

26. (i) Define activity of a radioactive material and write its SI unit.
 (ii) Plot a graph showing variation of activity of a given radioactive sample with time.
 (iii) The sequence of stepwise decay of a radioactive nucleus is

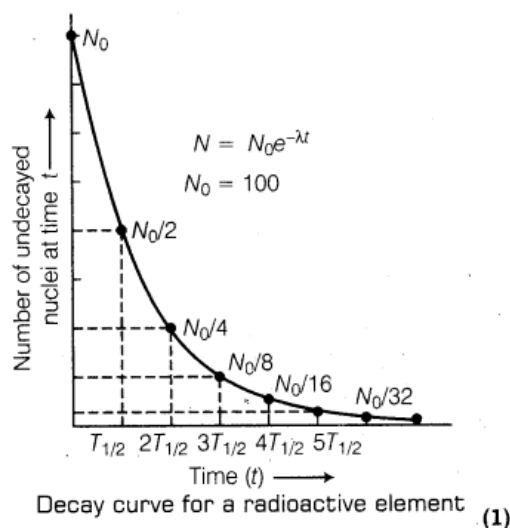


If the atomic number and mass number of D_2 are 71 and 176 respectively, what are their corresponding values for

[HOTS; Delhi 2010]

Ans.(i)

The curve representing the law of radioactive decay is shown as below:



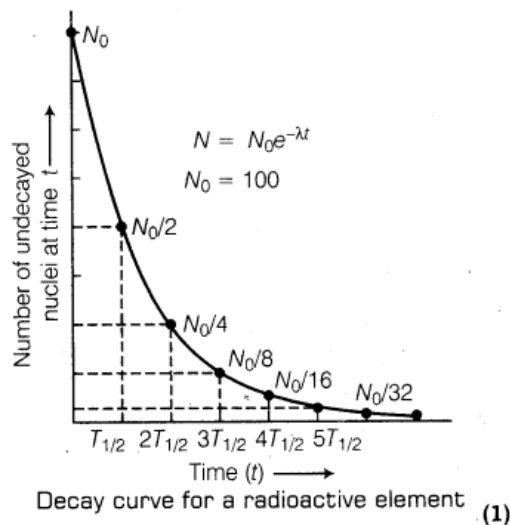
The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

The SI unit of activity is Becquerel (Bq).

$$1 \text{ Bq} = 1 \text{ disintegration /second} \quad (1)$$

(ii)


The curve representing the law of radioactive decay is shown as below:



The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

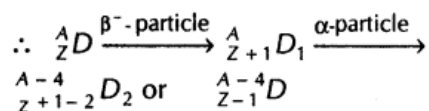
The SI unit of activity is Becquerel (Bq).

$$1 \text{ Bq} = 1 \text{ disintegration /second} \quad (1)$$

- (iii)  In these types of questions, remember the change in atomic number and mass number during the emission of α , β or γ .

In β -decay, the mass number remains same and atomic number increases by 1 unit. In α -decay, the mass number decreases by 4 units and atomic number decreases by 2 units.

Let mass and atomic number of D be A and Z , respectively.



According to the question, the mass number and atomic number of D_2 are 176 and 71, respectively.

(a) Atomic number of $D = Z - 1 = 71$

$$\Rightarrow Z = 72 \quad (1/2)$$

(b) Mass number of $D = A - 4 = 176$

$$\Rightarrow A = 176 + 4 = 180 \quad (1/2)$$

27. What is the basic mechanism for the emission of β^- and β^+ -particles in a nuclide? Give an example by writing explicitly a decay process for β -emission. Is

- the energy of the emitted β -particles continuous or discrete?
- the daughter nucleus obtained through β -decay, an isotope or an isobar of the parent nucleus?

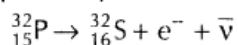
[Delhi 2010C]

Ans.

During β^- -decay from the nucleus, nucleus undergoes a change in such a way that atomic number increases by one and mass number remains same.

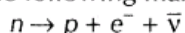
In β^+ -decay, the mass number of present radioactive nucleus remains same, whereas atomic number decreases by one. (1)

Example of β^- -decay



[Half-life = 26 days]

In β -decay, an electron and an antineutrino are created in the following manner.



- The energy of emitted β -particles is continuous. (1)

- (ii) As, there is no change in mass number during β -decay. So, the daughter nucleus is isobar of the parent nucleus. (1)

28. (i) What is meant by half-life of a radioactive element?
 (ii) The half-life of a radioactive substance is 30 s. Calculate
 (a) the decay constant and
 (b) time taken for the sample to decay by 3/4th of the initial value. [Foreign 2009]

Ans.

- (i) (a) **Half-life** Half-life of a radioactive element is defined as the time during which half the number of atoms present initially in the sample of the element decay or it is the time during which number of atoms left undecayed in the sample is half the total number of atoms present in the sample. It is represented by $T_{1/2}$.

From the equation $N = N_0 e^{-\lambda t}$,

$$\text{At half-life, } t = T_{1/2}, N = \frac{N_0}{2}$$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2$$

On taking log on both sides, we get

$$\lambda T_{1/2} = \log_e 2$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{\log_{10} 2 \times 2.303}{\lambda}$$

$$= \frac{0.3010 \times 2.303}{\lambda}$$

After n half-life, the the number of atoms

left undecayed is given by $N = N_0 \left(\frac{1}{2}\right)^n$

$$T_{1/2} = \frac{0.6932}{\lambda} \quad (1)$$

- (b) **Average Life** Average life of a radioactive element can be obtained by calculating the total life time of all atoms of the element and dividing it by the total number of atoms present initially in the sample of the element. Average life or mean life of radioactive element is

$$\tau = \frac{\text{Total life time of all atoms}}{\text{Total number of atoms}}$$

$$\tau = \int_0^{N_0} \frac{t dN}{N_0} = \int_0^\infty \frac{-\lambda N_0 e^{-\lambda t} dt \times t}{N_0}$$

[When $N = N_0, t = 0$ and when $N = 0, t = \infty$]

$$[\because dN = -\lambda(N_0 e^{-\lambda t}) dt]$$

$$\begin{aligned}
&= \lambda \int_0^\infty t e^{-\lambda t} dt = \lambda \left[\left\{ t \frac{e^{-\lambda t}}{-\lambda} \right\}_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right] \\
&= \lambda \left(0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right) \\
&= \int_0^\infty e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^\infty = 0 - \frac{1}{-\lambda} = \frac{1}{\lambda} \\
\tau &= \frac{1}{\lambda} = \frac{1}{0.6931/T_{1/2}} \\
\tau &= 1.44 T_{1/2} \quad (1)
\end{aligned}$$

(ii) Given, $\lambda = 0.3465 \text{ (day)}^{-1}$

According to the radioactive decay law, we have

$$\begin{aligned}
R &= R_0 e^{-\lambda t} \\
\Rightarrow \frac{R_0 \times 75}{100} &= R_0 e^{-0.3465t} \Rightarrow \frac{3}{4} = e^{-0.3465t} \\
\Rightarrow t &= 0.830 \text{ s} \quad (1)
\end{aligned}$$

(ii) $T_{1/2} = 30 \text{ s}$

(a) $\lambda = ?$

$$\begin{aligned}
\therefore T_{1/2} &= \frac{0.693}{\lambda} \\
\Rightarrow \lambda &= \frac{0.693}{T_{1/2}} = \frac{0.693}{30} = 0.0231 \text{ s}^{-1} \quad (1)
\end{aligned}$$

$$(b) \therefore N = N_0 \left(\frac{1}{2} \right)^n$$

where, n = number of half-lives

N = number of undisintegrated nuclei present in the sample

N_0 = original number of undisintegrated atom

undisintegrated atom

$$\text{Here, } N = N_0 - \frac{3}{4} N_0$$

$$N = \frac{1}{4} N_0 \Rightarrow N = N_0 \left(\frac{1}{2} \right)^n$$

$$\frac{N_0}{4} = N_0 \left(\frac{1}{2} \right)^n \Rightarrow \left(\frac{1}{2} \right)^2 = \left(\frac{1}{2} \right)^n$$

$$\Rightarrow n = 2$$

But number of half-lives

$$2 = \frac{\text{Total time taken}}{30 \text{ s}}$$

$$\text{Total time taken} = 60 \text{ s} = 1 \text{ min} \quad (1)$$

29. An observer in a laboratory starts with N_0 nuclei of a radioactive sample and keep on observing the number (N) of left over nuclei at regular intervals of 10 min each. She prepares the following table on the basis of her observation.

Time t (in min)	$\log_e \left(\frac{N_0}{N} \right)$
0	0
10	3.465
20	6.930
30	10.395
40	13.860

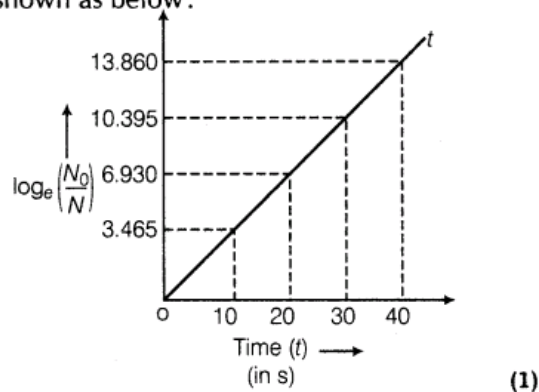
Use this data to plot a graph of $\log_e(N_0/N)$ vs time (t) and calculate the
 (i) decay constant and
 (ii) half-life of the given sample.

[Delhi 2009C]

Ans.

💡 We have to keep in mind these things while drawing the graph.
 The graph will be straight line passing through the origin. The slope of the graph will be constant.

The graph between $\log_e \left(\frac{N_0}{N} \right)$ and time is shown as below:



$$(i) \quad \therefore \log_e \left(\frac{N_0}{N} \right) = \lambda t$$

\Rightarrow Slope of $\log_e \left(\frac{N_0}{N} \right)$ vs time t graph gives

decay constant λ .

$$\therefore \lambda = \frac{3.465}{10} \text{ s}^{-1}$$

[using observation given in table]

$$= \frac{6.930}{20}$$

$$= 0.3465 \quad (1)$$

$$(ii) \quad \therefore \text{Half-life, } T_{1/2} = \frac{0.693}{\lambda}$$

$$T_{1/2} = \frac{0.693}{0.3465} = 2 \text{ s}$$

$$\text{Half-life, } T_{1/2} = 2 \text{ s} \quad (1)$$

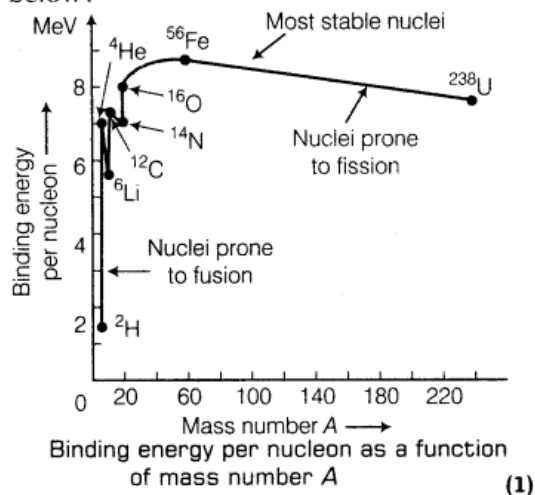
Mass Defect and Binding Energy

2 Marks Questions

1. Using the curve for the binding energy per nucleon as a function of mass number A , state clearly how the release in energy in the processes of nuclear fission and nuclear fusion can be explained. [All India 2011]

Ans.

The binding energy per nucleon curve is shown as below:



Explanation of Release of Energy in Nuclear Fission and Fusion The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei are less stable than middle level nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass). Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission

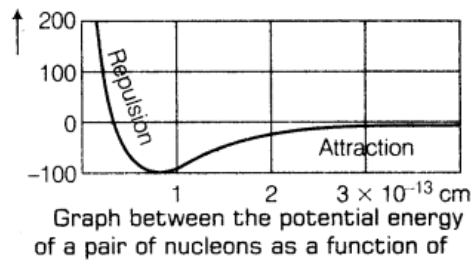
In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value of binding energy per nucleon to high value and release of energy takes place in fusion e.g., two ${}_1\text{H}^2$ (Binding energy per nucleon ≈ 1.5 MeV/nucleon) combine to form ${}_2\text{He}^4$ (Binding energy per nucleon ≈ 7 MeV/nucleon) and therefore the energy is liberated during nuclear fusion.

(1/2)

2. Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions which you can draw regarding the nature of nuclear forces. [All India 2010]

Ans.

Graph manifests that



(1)

The conclusions drawn from the graph are given as below :

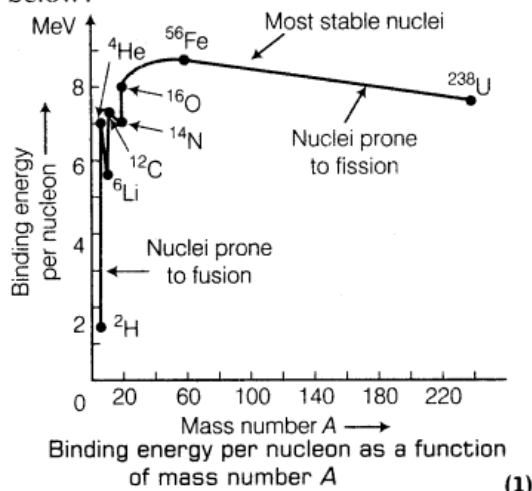
- (i) Nuclear force is a short range force.
- (ii) Nuclear force is of attractive nature when separation between the nuclei greater than 1 fm and of repulsive nature when separation is less than 1 fm.

(1)

3. Draw a plot of the binding energy per nucleon as a function of mass number for a large number of nuclei $20 < A < 240$. How do you explain the constancy of binding energy per nucleon in the range of $30 < A < 170$ using the property that nuclear force is short-ranged? [All India 2010]

Ans.

The binding energy per nucleon curve is shown as below:



(1)


Explanation of Release of Energy in Nuclear Fission and Fusion The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei are less stable than middle level nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass). Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission

In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value of binding energy per nucleon to high value and release of energy takes place in fusion e.g., two ${}_1\text{H}^2$ (Binding energy per nucleon ≈ 1.5 MeV/nucleon) combine to form ${}_2\text{He}^4$ (Binding energy per nucleon ≈ 7 MeV/nucleon) and therefore the energy is liberated during nuclear fusion. (1/2)

The binding energy per nucleon in the range of $30 < A < 170$ has average binding energy per nucleon = 8.5 MeV. The higher value of binding energy per nucleon is due to stability of these nucleons. Neutron-proton ratio is higher in this range of mass number which leads to stability of the nuclei. Also, the nuclear force is strongly attractive enough to overcome the coulombian repulsive force acting between positively charged protons,

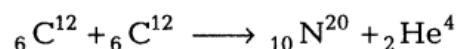
4. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV is splitted into two fragments Y and Z of mass numbers 110 and 130. The binding energy of nucleons in Y and Z is 8.5 MeV per nucleon. Calculate the energy released per fission in MeV. [hots; Delhi 2010]

Ans.

 In these types of questions, we have to keep in mind about difference of mass involved between reactants and product. Energy will be involved accordingly.

$$\begin{aligned} \text{Energy released per fission} \\ &= (110 + 130) \times 8.5 \text{ MeV} - 240 \times 7.6 \text{ MeV} \quad (1) \\ &= 240 \times (8.5 - 7.6) \text{ MeV} \\ &= 240 \times 0.9 = 216.0 \text{ MeV} \quad (1) \end{aligned}$$

5. If both the numbers of protons and neutrons are conserved in a nuclear reaction like



In what way, is the mass converted into the energy? Explain. [Delhi 2010]

Ans.

The sum of masses of nuclei of product element is less than the sum of masses of reactants and hence, loss of mass takes place during the reaction. This difference of mass of product element and reactant converts into energy and liberated in the form of heat. (1)

Here, sum of masses of ${}_{10}\text{N}^{20}$ and ${}_2\text{He}^4$ is less than the sum of two ${}_6\text{C}^{12}$ and conversion of this mass defect is used to produce energy. (1)

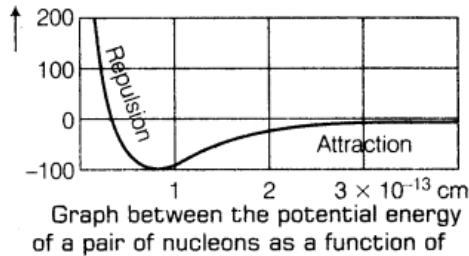
6.(i) The mass of a nucleus in its ground state is always less than the total mass of its constituents neutrons and protons. Explain, (ii) Plot a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. [All India 2009]

Ans.

- (i) Mass defect occurs in nucleus which converts into energy as per Einstein's mass-energy relation, $E = mc^2$ and produces binding energy. This energy binds nucleons together due to nuclear forces in spite of repulsive coulombian forces. (1)

(ii)

Graph manifests that

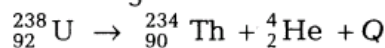


(1)

The conclusions drawn from the graph are given as below :

- (i) Nuclear force is a short range force.
(ii) Nuclear force is of attractive nature when separation between the nuclei greater than 1 fm and of repulsive nature when separation is less than 1 fm. (1)

7. Calculate the energy released in the following nuclear reaction :



[Mass of ${}_{92}^{238}\text{U} = 238.05079 \text{ u}$

Mass of ${}_{90}^{234}\text{Th} = 234.043630 \text{ u}$

Mass of ${}_2^4\text{He} = 4.002600 \text{ u}$
 $1 \text{ u} = 931.5 \text{ MeV}$ [All India 2008]

Ans.

Sum of masses of ${}_{90}^{234}\text{Th}$ and ${}_2^4\text{He}$
 $= 234.043630 + 4.002600$
 $= 238.046230 \text{ u}$

Mass of ${}_{92}^{238}\text{U} = 238.05079 \text{ u}$ (1)

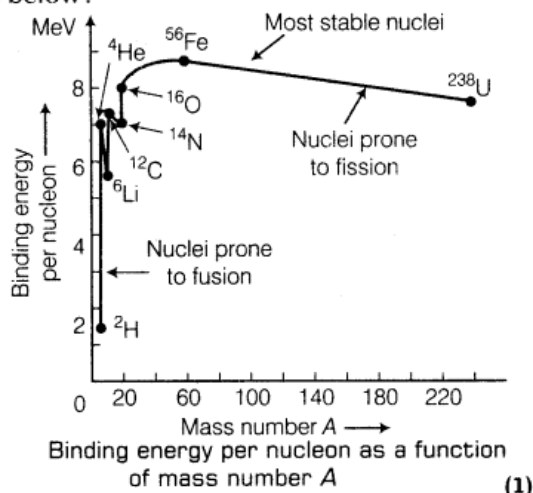
\therefore Loss of mass (mass defect) in given nuclear reaction
 $= 238.05079 - 238.046230$
 $= 0.00456 \text{ u}$

\therefore Energy released in nuclear reaction
 $= 0.00456 \times 931.5 \text{ MeV}$
 $= 4.24764 \text{ MeV}$ (1)

8. Sketch a graph showing the variation of binding energy per nucleon as a function of mass number A for large number of nuclei. State briefly from which region of the graph can release of energy in the process of nuclear fusion be explained. [Foreign 2008]

Ans.

The binding energy per nucleon curve is shown as below:



Explanation of Release of Energy in Nuclear Fission and Fusion The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei are less stable than middle level nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass). Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission

In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value of binding energy per nucleon to high value and release of energy takes place in fusion e.g., two ${}_1\text{H}^2$ (Binding energy per nucleon ≈ 1.5 MeV/nucleon) combine to form ${}_2\text{He}^4$ (Binding energy per nucleon ≈ 7 MeV/nucleon) and therefore the energy is liberated during nuclear fusion. (1/2)

In the range of mass number, 2 to 20, these are maximum and minimum as the curve. Here ${}_2\text{He}^4$, ${}_6\text{C}^{12}$ and ${}_8\text{O}^{16}$ are at maxima and ${}_1\text{H}^2$, Li, N are at minima. This range of mass number may facilitate release of energy in nuclear fusion, e.g. two ${}_1\text{H}^2$ nuclei of low binding energy when combined in a nuclear fusion to form ${}_2\text{He}^4$ of high value of binding energy per nucleon. In this process, energy will release in the form of heat. (2)

9. Why is it necessary to slow down the neutrons produced through the fission of ${}_{92}^{235}\text{U}$ nuclei (by neutrons) to sustain a chain reaction? What type of nuclei are (preferably) needed for slowing down fast neutrons? [All India 2008C]

Ans.

Average kinetic energy of neutrons produced in nuclear fission of ${}_{92}^{235}\text{U}$ is nearly 2 MeV, whereas the chances of absorption of neutrons of average kinetic energy of nearly 0.024 MeV is high by uranium nuclei. So, there is a need to slow down

the fast neutrons using appropriate substance namely moderator into slow thermal neutrons. Nuclei which have comparable mass to that of neutrons should be preferable be used to slow down fast neutrons. It is due to the fact that the elastic collision between fast neutrons and slow moving protons lead to interchange the velocities

- 10. (i)** Write the relation for Binding Energy (BE) (in MeV) of a nucleus of mass ${}^A_Z M$, atomic number (Z) and mass number (A) in terms of the masses of its constituents namely neutrons and protons.
- (ii)** Draw a plot of BE/A versus mass number A for $2 \leq A \leq 170$. Use this graph to explain the release of energy in the process of nuclear fusion of two light nuclei. [Delhi 2014]

Ans.

$$(i) \text{ BE } = [Zm_p + (A - Z)m_n - {}^A_Z M] \times c^2$$

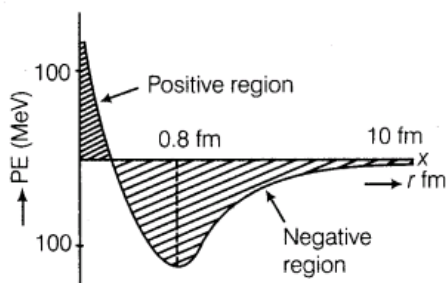
where, M is mass of nucleus, m_p is the mass of proton and m_n is the mass of neutron.

11. Draw a plot of potential energy between a pair of nucleons as a function of their separation. Mark the regions where potential energy is

- (i) positive and
(ii) negative. [Delhi 2013]

Ans.

Plot between the potential energy of a pair of nucleons as a function of their separation (1)



- (i) For distance less than 0.8 fm, negative PE decreases to zero and then becomes positive.

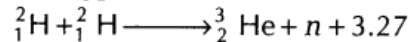
12. Answer the following.

- (i) Why is the binding energy per nucleon found to be constant for nuclei in the range of mass number (A) lying between 30 and 170?
- (ii) When a heavy nucleus with mass number $A = 240$ breaks into two nuclei, $A = 120$, energy is released in the process. [Foreign 2012]

Ans.

- (i) The binding energy per nucleon for nucleus of range, $30 < A < 170$ is close to its maximum value. So, the nucleus belongs to this region is highly stable and does not show radioactivity. (1)
- (ii) Binding energy per nucleon is smaller for heavier nuclei than the middle ones, i.e. heavier nuclei are less stable. When a heavier nucleus such as nucleus of mass number 240 splits into lighter nuclei (mass number 120), the BE/nucleon changes from about 7.6 MeV to 8.4 MeV. Greater BE of the product nuclei result in the liberation of energy. (2)

13. (i) In a typical nuclear reaction, e.g.



although number of nucleons is conserved yet energy is released. How? Explain.

(ii) Show that nuclear density in a given nucleus is independent of mass number A .

Ans.

- (i) In a nuclear reaction, the sum of the masses of the target nucleus (${}^2_1\text{H}$) and the bombarding particle (${}^2_1\text{H}$) may be greater or less than the sum of the masses of the product nucleus (${}^3_2\text{He}$) and the outgoing particle (${}_0^1n$). So, from the law of conservation of mass-energy, some energy (3.27 MeV) is evolved or involved in a nuclear reaction. This energy is called Q -value of the nuclear reaction. (1)

- (ii) Density of nuclear matter is the ratio of mass of the nucleus and its volume.

Density of the nuclear matter

$$= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} \quad \dots(i)$$

If m is average mass of a nucleon and R is the nuclear radius, then mass of nucleus = mA , where A is the mass number of the element.

$$\text{Volume of the nucleus} = \frac{4}{3} \pi R^3$$

$$= \frac{4}{3} (\pi R_0 A^{1/3})^3 = \frac{4}{3} \pi R_0^3 A$$

$$\text{Thus, density of nucleus} = \frac{mA}{\frac{4}{3} \pi R_0^3 A}$$

where, m = mass of one nucleon

A = mass

$$\text{Number} = \frac{3m}{4\pi R_0^3} \quad (1)$$

As, m and R_0 are constants, therefore density of the nuclear matter is the same for all elements. Now, using $m = 1.66 \times 10^{-27} \text{ kg}$.

Substituting the value of Eq. (ii) and Eq. (iii) in Eq. (i), we get

$$\begin{aligned} &= \frac{A \times 1.66 \times 10^{-27}}{\left(\frac{4}{3} \pi R_0^3\right) A} \\ &= \frac{1.66 \times 10^{-27}}{\left(\frac{4}{3} \pi R_0^3\right)} \end{aligned}$$

which shows that the density is independent of mass number A .

$$\begin{aligned} \text{Using } R_0 &= 1.1 \times 10^{-15} \text{ m and density} \\ &= 2.97 \times 10^{17} \text{ kg m}^{-3} \end{aligned}$$

14.(i) What characteristic property of nuclear force explains the constancy of binding energy per nucleon (BE/A) in the range of mass number A lying $30 < A < 170$?

(ii) Show that the density of nucleus over a wide range of nuclei is constant and independent of mass number A . [Delhi 2012]

Ans. (i) The saturation effect of nuclear force explains the constancy of BE/A over wide range of mass number, $70 > A > 30$. Saturation effect implies that nucleon interacts only with its neighbouring nucleons and does not interact with nucleons which are not in direct contact with it.

(ii) Density of nuclear matter is the ratio of mass of the nucleus and its volume.

$$\begin{aligned} \text{Density of the nuclear matter} \\ &= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} \quad \dots(i) \end{aligned}$$

If m is average mass of a nucleon and R is the nuclear radius, then mass of nucleus = mA , where A is the mass number of the element.

$$\begin{aligned} \text{Volume of the nucleus} &= \frac{4}{3} \pi R^3 \\ &= \frac{4}{3} (\pi R_0 A^{1/3})^3 = \frac{4}{3} \pi R_0^3 A \end{aligned}$$

$$\text{Thus, density of nucleus} = \frac{mA}{\frac{4}{3} \pi R_0^3 A}$$

where, m = mass of one nucleon

A = mass

$$\text{Number} = \frac{3m}{4\pi R_0^3} \quad (1)$$

As, m and R_0 are constants, therefore density of the nuclear matter is the same for all elements. Now, using $m = 1.66 \times 10^{-27} \text{ kg}$.

Substituting the value of Eq. (ii) and Eq. (iii) in Eq. (i), we get

$$\begin{aligned} &= \frac{A \times 1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi R_0^3\right)A} \\ &= \frac{1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi R_0^3\right)} \end{aligned}$$

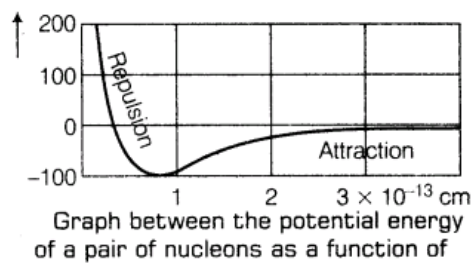
which shows that the density is independent of mass number A .

$$\begin{aligned} \text{Using } R_0 &= 1.1 \times 10^{-15} \text{ m and density} \\ &= 2.97 \times 10^{17} \text{ kg m}^{-3} \end{aligned}$$

15. Draw a plot of potential energy of a pair of nucleons as a function of their separations. Mark the regions where the nuclear force is (i) attractive and (ii) repulsive. Write any two characteristic features of nuclear forces. [All India 2012]

Ans.

Graph manifests that



(1)

The conclusions drawn from the graph are given as below :

- (i) Nuclear force is a short range force.
- (ii) Nuclear force is of attractive nature when separation between the nuclei greater than 1 fm and of repulsive nature when separation is less than 1 fm. (1)

Net interactive force is zero when PE is minimum, i.e. nearly, $r_0 = 1 \text{ fm}$ (in graph).

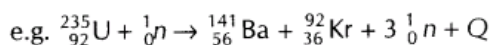
- (i) The nuclear force is attractive when separation between the nuclei is greater than $r_0 > 1 \text{ fm}$. (1)
- (ii) Repulsive when $r_0 < 1 \text{ fm}$. (1)

16. Explain giving necessary reactions, how energy is released during (i) fission (ii) fusion [All India 2011 c]

Ans.

- (i) **Nuclear Fission** The phenomenon of splitting of heavy nuclei (mass number > 120) into smaller nuclei of nearly equal masses is known as nuclear fission.

In nuclear fission, the sum of the masses of the product is less than the sum of masses of the reactants. This difference of mass gets converted into energy $E = mc^2$ and hence sample amount of energy is released in a nuclear fission.



Masses of reactant

$$\begin{aligned} &= 235.0439 \text{ amu} + 1.0087 \text{ amu} \\ &= 236.0526 \text{ amu} \end{aligned}$$

Masses of product

$$\begin{aligned} &= 140.9139 + 91.8973 + 3.0261 \\ &= 235.8373 \text{ amu} \end{aligned}$$

$$\text{Mass defect} = 236.0526 - 235.8373$$

$$= 0.2153 \text{ amu}$$

$$\therefore 1 \text{ amu} \equiv 931 \text{ MeV}$$

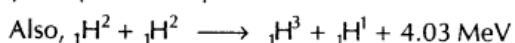
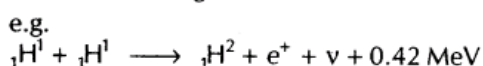
$$\begin{aligned} \Rightarrow \text{Energy released} &= 0.2153 \times 931 \\ &= 200 \text{ MeV nearly} \end{aligned}$$

Thus, energy is liberated in nuclear fission

if ${}_{92}^{235}\text{U}$. (1)

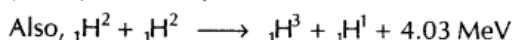
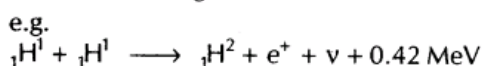
- (ii) **Nuclear Fusion** The phenomenon of conversion of two lighter nuclei into a single heavy nucleus is called nuclear fusion. (1/2)

Since, the mass of the heavier product nucleus is less than the sum of masses of reactant nuclei and therefore certain mass defect occurs which converts into energy as per Einstein's mass-energy relation. Thus, energy is released during nuclear fusion.



- (ii) **Nuclear Fusion** The phenomenon of conversion of two lighter nuclei into a single heavy nucleus is called nuclear fusion. (1/2)

Since, the mass of the heavier product nucleus is less than the sum of masses of reactant nuclei and therefore certain mass defect occurs which converts into energy as per Einstein's mass-energy relation. Thus, energy is released during nuclear fusion.



5 Marks Questions

- 17.(i) Draw the plot of binding energy per nucleon (BE/A) as a function of mass number A. Write two important conclusions that Can be drawn regarding the nature of nuclear force.
(ii) Use this graph to explain the release of energy in both the processes of nuclear fusion and fission.



- (iii) Write the basic nuclear process of neutron undergoing β -decay. Why is the detection of neutrinos found very difficult? [HOTS; All India 2013]

Ans.

While drawing the plot, we have to keep in mind that first binding energy will increase sharply and then it will be constant almost.

- (i) For plot of binding energy per nucleon as the function of mass number A

Refer to ans 1.

(1)

Following are the two conclusions that can be drawn regarding the nature of the nuclear force.

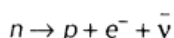
- (a) The force is attractive and strong enough to produce a binding energy of few MeV per nucleon. (1/2)
- (b) The constancy of the binding energy in the range of $30 < A < 170$ is a consequence of the fact that the nuclear force is short range for (1/2)

- (ii) **Nuclear Fission** A very heavy nucleus (say $A = 240$) has lower binding energy per nucleon as compared to the nucleus with $A = 120$. Thus, if the heavier nucleus breaks into the lighter nucleus with high binding energy per

nucleon, nucleons are tightly bound. This implies that energy will be released in the process which justifies the energy released in fission reaction

Nuclear Fusion When two light nuclei ($A < 10$) are combined to form a heavier nuclei, the binding energy of the fused heavier nuclei is more than the binding energy per nucleon of the lighter nuclei. Thus, the final system is more tightly bound than the initial system. Again the energy will be released in fusion reaction.

- (iii) The basic nuclear process of neutron undergoing β -decay is given as below:



Neutrinos interact very weakly with matter so, they have a very high penetrating power. That's why the detection of neutrinos is found very difficult. (1)

18. Define the Q-value of a nuclear process. When can a nuclear process not proceed spontaneously? If both the number of protons and the number of neutrons are conserved in a nuclear reaction in what way is mass converted into energy (or vice-versa) in the nuclear reaction? [All India 2010 c]

Ans.

The Q-value of a nuclear process refers the energy release in the nuclear process which can be determined using Einstein's mass-energy relation, $E = mc^2$. The Q-value is equal to the difference of mass of products and reactant nuclei multiplied by square of velocity of light. (2)

The nuclear process does not proceed spontaneously when Q-value of a process is negative or sum of masses of product is greater than sum of masses of reactant.

19. Draw a plot of binding energy per nucleon (BE/A) versus mass number (A) for a large number of nuclei lying between $2 < A < 240$. Using this graph, explain clearly how the energy is released in both the process of nuclear fission and fusion? [All India 2009 C]

Ans.

While drawing the plot, we have to keep in mind that first binding energy will increase sharply and then it will be constant almost.

(i) For plot of binding energy per nucleon as the function of mass number A

Refer to ans 1.

(1)

Following are the two conclusions that can be drawn regarding the nature of the nuclear force.

(a) The force is attractive and strong enough to produce a binding energy of few MeV per nucleon.

(1/2)

(b) The constancy of the binding energy in the range of $30 < A < 170$ is a consequence of the fact that the nuclear force is short range for

(1/2)

(ii) **Nuclear Fission** A very heavy nucleus (say $A = 240$) has lower binding energy per nucleon as compared to the nucleus with $A = 120$. Thus, if the heavier nucleus breaks into the lighter nucleus with high binding energy per

nucleon, nucleons are tightly bound. This implies that energy will be released in the process which justifies the energy released in fission reaction

Nuclear Fusion When two light nuclei ($A < 10$) are combined to form a heavier nuclei, the binding energy of the fused heavier nuclei is more than the binding energy per nucleon of the lighter nuclei. Thus, the final system is more tightly bound than the initial system. Again the energy will be released in fusion reaction.